ex plainly.

~~Geometrical~~ Co-ordinate and Degree of freedom

Generalised

Problem (ii) paper XIII unit III classical mechanics Dr. ... Bisht

Suppose a dynamical system consist of N particles P_i with co-ordinates x_i, y_i, z_i.

Let the dynamical system be subjected to d finite constraints

$$F_\alpha(t, x_i, y_i, z_i) = 0 \quad \begin{pmatrix} \alpha = 1, 2, 3, \ldots \\ i = 1, 2, 3, \ldots \end{pmatrix}$$

Here constraints show that the dynamical system of the 3N quantities may be expressed in terms of remaining 3N-d quantities.

If $3N-d=n$, then there are n quantities to be independent. Here independent n quantities which specify the positions of the particles of the system, are called generalised co-ordinate.

Generally, the generalised co-ordinates are denoted by q_i ($i=1,2,\ldots,n$). If the dynamical system is non-holonomic then in practice it is not possible to solve non-integrable differential constraints.

Let there be d finite constraints and g non-integrable differential constraints. Then 3N-d will be generalised co-ordinate with g non-integrable differential constraint. In this case the degree of freedom is $3N-d+g$.

If the system is holonomic with d finite constraint and g integrable differential constraints, then in this case there will be $3N-d-g$ generalised co-ordinate and the degree of freedom is $3N+d+g$.

If there are only d finite constraint then in this case the number of generalised co-ordinate is equal to the degree of freedom.

Ex 1. Let a particle be constrained to move on the circumference of a circle of radius r.

To find the position of particle at time t. we need two Cartesian Co-ordinates x and y. In polar Co-ordinates we need two Co-ordinates r and θ. But here r is fixed so if we only know θ, then the position of the particle may be known. So here θ is the generalised Co-ordinate of the system.

Ex 2. Let a particle be constrained to move on the surface of a sphere. To know the position of the particle at time t.

We need three Cartesian Co-ordinates x, y and z. In spherical Co-ordinate system we need three Co-ordinates r, θ and ϕ. But r (radius of sphere) is fixed. So to know the position of the particle we need only two Co-ordinates θ and ϕ. Here θ and ϕ are called the generalised Co-ordinates of the system. It is obvious that

$$x = x(\theta, \phi), \; y = y(\theta, \phi), \; z = z(\theta, \phi)$$

In general if q_1, q_2, ---, q_n be the generalised Co-ordinates of the particles of the system, then

$$r_\nu = r_\nu (q_1, q_2, ---, q_n, t)$$